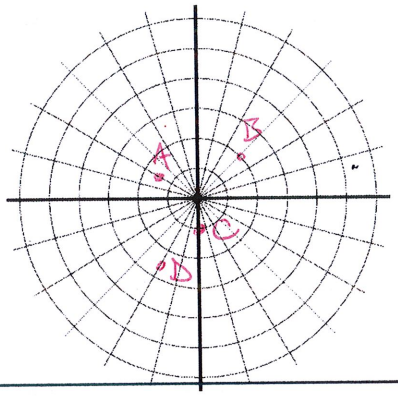
GSE PreCalculus

Unit 8: Polar Functions Review

Key

Graph the following polar points:

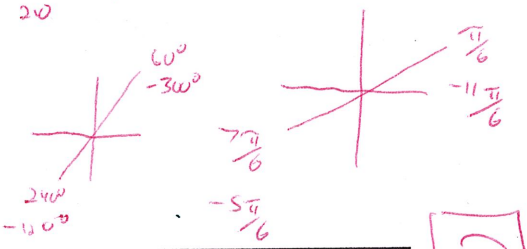
1. A $\left(1.5, -\frac{7\pi}{6}\right)$
2. B $\left(-2, -135^\circ\right)$
3. C $\left(-1, \frac{\pi}{2}\right)$
4. E $\left(2.5, 240^\circ\right)$



Find four different pairs of polar coordinates that name the given point if:

$-360^\circ \le \sigma \le 360^\circ$ or $-2\pi \le \sigma \le 2\pi$

5. $\left(2, -150^\circ\right)$ $(2, 210^\circ)$, $(-2, -330^\circ)$, $(-2, 30^\circ)$
6. $\left(5, 240^\circ\right)$ $(5, -120^\circ)$, $(-5, 60^\circ)$, $(-5, -300^\circ)$
7. $\left(2, \frac{\pi}{6}\right)$ $(2, -11\pi/6)$, $(-2, 7\pi/6)$, $(-2, -5\pi/6)$



Rectangular Coordinates to Polar Coordinates:

8. (8,10) 8) $r = \sqrt{8^2+10^2} = 12.81$ $\theta = \tan^{-1}\left(\frac{10}{8}\right) = 51.34^\circ$ $(12.81, 51.34^\circ)$
9. (-9, -4) 9) $r = \sqrt{(-9)^2+(-4)^2} = 9.85$ $\theta = \tan^{-1}\left(\frac{-4}{-9}\right) + 180 = 203.96$ $(9.85, 203.96^\circ)$

Polar Coordinates to Rectangular Coordinates:

10. (3, -120°) 10) $(3\cos -120^\circ, 3\sin -120^\circ)$ $(-1.5, -2.60)$
11. (-2, 135°) 11) $(-2\cos 135^\circ, -2\sin 135^\circ)$ $(1.41, -1.41)$

1. Convert from rectangular equations to polar equations:

12. $y = \sqrt{3}x$ $\theta = 60^\circ$
13. $x^2 + (y-3)^2 = 9$ $r = 6\sin\theta$
14. $x^2 - y^2 = 1$ $r^2 = \sec\theta$

Convert from polar equations to rectangular equations. Then, identify the resulting figure.

13. $r = 10$ $x^2 + y^2 = 100$
14. $\theta = -\frac{\pi}{3}$ $y = -\sqrt{3}x$
15. $r = 2\cos\theta$ $(x-1)^2 + y^2 = 1$
16. $r = \frac{1}{\cos\theta + \sin\theta}$ $y = -x + 1$

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Represent complex numbers (polar form) & complex number operations:

17. Explain how you would represent $-3-i$ on the complex plane.

Move 3 to the left and 1 down from the origin

18. Find the conjugate of $-4+2i$.

$-4-2i$

19. $\dfrac{6-i}{-4+2i}$ $\quad -\frac{13}{10}-\frac{2}{5}i$

20. $(-1+4i)-(2+7i)$ $\quad -3-3i$

21. $(2-i)(3+4i)$ $\quad 10+5i$

Express each complex number in polar form:

22. $-2+5i$ $\quad (5.39, 111.80°)$

23. $6+2i$ $\quad (6.32, 18.43°)$

Convert the polar form of a complex number to its rectangular form:

24. $z=4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ $\quad 2+3.46i$

25. $z=5\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ $\quad -3.54+3.54i$

Cumulative Review:

26. Solve: $\begin{aligned} x-2y&=7 \\ 4x+5y&=-2 \end{aligned}$ $\quad \begin{bmatrix} 31/13 \\ -30/13 \end{bmatrix}$

27. Find the standard form equation of $2x^2-4y^2-6x+8y-10=2$ $\quad \frac{(x-1.5)^2}{6.25}-\frac{(y-1)^2}{3.125}=1$

28. Evaluate: $\sin\left(\frac{4\pi}{3}\right)$ $\quad -\frac{\sqrt{3}}{2}$

29. Evaluate: $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\quad 150°$

30. Find $\sec\theta$, if the angle is in Quadrant II, $\sin\theta=\frac{2}{5}$

$\cos\theta=\frac{\sqrt{21}}{5}$ $\quad \sec\theta=\frac{5}{\sqrt{21}}=\frac{5\sqrt{21}}{21}$

31. Evaluate $\begin{vmatrix} -5 & 1 \\ -2 & -2 \end{vmatrix}$ $\quad = 12$

32. Find the asymptotes of $y=\tan(2x-60)$ $\quad x=-15°, 30°$

33. Solve: $4\sin^2 x-3=0$ $\quad x=60°$

34. Find $< E, given\ that\ r=6, b=2, e=5$

35. Find the component form of the vector, $given\ \|5\|, \emptyset = 48°$ $\quad \langle 5\cos 48, 5\sin 48\rangle$

$\langle 3.35, 3.72\rangle$

36. An airplane is raveling 300 kilometers per hour due east. A wind is blowing 35 kilometers per hour S 75°W. What is the resulting velocity of the airplane?

Speed = 266.34 kph $\quad \theta=-1.95°$ or $358.05°$

12) $y = \sqrt{3}x$

$\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{3}\,r\cos\theta}{r\cos\theta}$

$\tan\theta = \sqrt{3}$

$\tan^{-1}(\tan\theta) = \tan^{-1}\sqrt{3}$

$\theta = 60°$

13) $x^2 + (y-3)^2 = 9$

$(r\cos\theta)^2 + (r\sin\theta - 3)^2 = 9$

$r^2\cos^2\theta + r^2\sin^2\theta - 6r\sin\theta + 9 = 9$

$r^2\cos^2\theta + r^2\sin^2\theta - 6r\sin\theta = 0$

$r^2\cos^2\theta + r^2\sin^2\theta = 6r\sin\theta$

$r\cos^2\theta + r\sin^2\theta = 6\sin\theta$

$r(\cos^2\theta + \sin^2\theta) = 6\sin\theta$

$r(1) = 6\sin\theta$

$r = 6\sin\theta$

14) $x^2 - y^2 = 1$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2(\cos 2\theta) = 1$

$r^2 = \frac{1}{\cos 2\theta}$

$r^2 = \sec\theta$

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13) $r = 10$

$r^2 = 100$

$x^2 + y^2 = 100$

14) $\theta = -\frac{\pi}{3}$

$\tan\theta = \tan\left(-\frac{\pi}{3}\right)$

$\frac{y}{x} = -\sqrt{3}$

$y = -\sqrt{3}x$

15) $r = 2\cos\theta$

$r^2 = 2r\cos\theta$

$x^2 + y^2 = 2x$

$x^2 - 2x + y^2 = 0$

$x^2 - 2x + 1 + y^2 = 1$

$(x-1)^2 + y^2 = 1$

16) $r = \frac{1}{\cos\theta + \sin\theta}$

$\frac{1}{r} = \frac{1}{r} \cdot \frac{1}{\cos\theta + \sin\theta}$

$1 = \frac{1}{r\cos\theta + r\sin\theta}$

$1 = \frac{1}{x + y}$

$x + y = 1$

$y = -x + 1$

19) $\frac{6-i}{-4+2i} \cdot \frac{-4-2i}{-4-2i} = \frac{-24-12i+4i+2i^2}{16+8i-8i-4i^2} = \frac{-24-8i-2}{16+4} = \frac{-26-8i}{20} = \boxed{-\frac{13}{10} - \frac{2}{5}i}$

21) $(2-i)(3+4i)$
$6+8i-3i-4i^2$
$6+5i+4$
$10+5i$

22) $r = \sqrt{(-2)^2+5^2} = 5.39$
$\theta = \tan^{-1}\left(\frac{5}{-2}\right)+180 = 111.80°$

23) $r = \sqrt{6^2+2^2} = 6.32$
$\theta = \tan^{-1}\left(\frac{2}{6}\right) = 18.43$

24) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$
$4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$
$2 + \frac{4\sqrt{3}}{2}i$
$2 + 3.46i$

25) $5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
$5\left(\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$
$-\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$
$-3.54 + 3.54i$

26) $2x^2 - 4y^2 - 6x + 8y - 10 = 2$
$2x^2 - 2y^2 - 3x + 4y - 5 = 1$
$x^2 - 3x - 2y^2 + 4y = 6$
$(x^2 - 3x + 2.25) - 2(y^2 - 2y + 1) = 6 + 2.25 - 2$
$\frac{(x-1.5)^2}{2(2.25)} - \frac{2(y-1)^2}{2(2.25)} = \frac{6.25}{2(2.25)}$
$\frac{(x-1.5)^2}{2} - (y-1)^2 = 3.125$
$\frac{(x-1.5)^2}{6.25} - \frac{(y-1)^2}{3.125} = 1$

Vertices = (-1, 1)
(4, 1)

32) $2x - 60 = -90$
$2x = -30$
$x = -15 + \frac{90}{2} = 30°$
1st 2nd

33) $4\sin^2 x - 3 = 0$
$4\sin^2 x = 3$
$\sin^2 x = \frac{3}{4}$
$\sin x = \frac{\sqrt{3}}{2}$
$x = 60°$

36) plane = $\langle 300\cos 0°, 300\sin 0° \rangle = \langle 300, 0 \rangle$
wind = $\langle 35\cos 195°, 35\sin 195° \rangle = \langle -33.81, -9.06 \rangle$
total = $\langle 266.19, -9.06 \rangle$

Speed = $\sqrt{266.19^2 + (-9.06)^2} = 266.34$ kph
$\theta = \tan^{-1}\left(\frac{-9.06}{266.19}\right) = -1.95°$