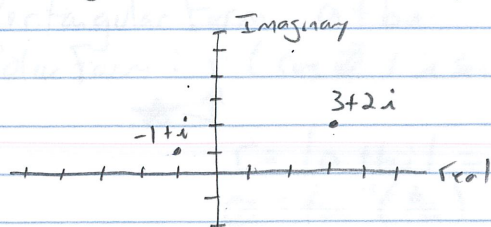


15-3

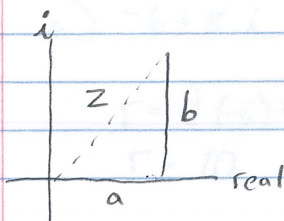
Complex Numbers & Polars

Rectangular Form of a Complex # : $a + bi$
↑ real ↑ imaginary



a) $3+2i \Rightarrow (3, 2)$
 b) $-1+i \Rightarrow (-1, 1)$

Absolute Value of a Complex Number



$$|a + bi| \Rightarrow z^2 = a^2 + b^2$$

$$z = \sqrt{a^2 + b^2}$$

$$|a + bi| = \sqrt{a^2 + b^2}$$

$$|4 - 3i| = \sqrt{(4)^2 + (-3)^2} = 5$$

Complex Numbers

$$i^1 = i \quad i^5 = i$$

$$i^2 = -1 \quad i^6 = -1$$

$$i^3 = -i \quad \text{and so}$$

$$i^4 = 1 \quad \text{further}$$

$$(2 + 3i) - (1 - 5i) = 2 + 3i - 1 + 5i = \boxed{1 + 8i}$$

$$(i)(2 - i) = 2i - i^2 = 2i - (-1) = \boxed{1 + 2i}$$

$$(2 - i)(3 + 4i) = 6 + 8i - 3i - 4i^2 = 6 + 8i - 3i + 4 = \boxed{10 + 5i}$$

by definition, $i = \sqrt{-1}$

$$\frac{3+2i}{-2+5i} \cdot \frac{-2-5i}{-2-5i} = \frac{-6-15i-4i-10i^2}{4+10i-10i-25i^2} = \frac{-6-19i+10}{4+25} = \boxed{\frac{4-19i}{29}}$$

↑ complex

conjugate

(change the sign on i)

↑ what happened to i?

(cont)

When this is 1 semester skill ✓ today

Converting Complex to Polar

Rectangular Form: $a+bi$

Polar Form: $r(\cos \theta + i \sin \theta) \Rightarrow r \text{ cis } \theta$ ↙ short hand

★ ↑

$$r = |a+bi| = \sqrt{a^2+b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) \text{ or } \tan^{-1}\left(\frac{b}{a}\right) + 180^\circ$$

} Look familiar?

#1) Express in Polar Form:

a) $-6+8i$

$$r = \sqrt{(-6)^2 + (8)^2}$$

$$r = 10$$

$$\theta = \tan^{-1}\left(\frac{8}{-6}\right) + 180^\circ$$

$$\theta = 126.87^\circ$$

$$10(\cos 126.87^\circ + i \sin 126.87^\circ)$$

$$\underline{\underline{\text{or } 10 \text{ cis } 126.87^\circ}}$$

b) $4+i\sqrt{3}$

$$r = \sqrt{(4)^2 + (\sqrt{3})^2}$$

$$r \approx 4.36$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

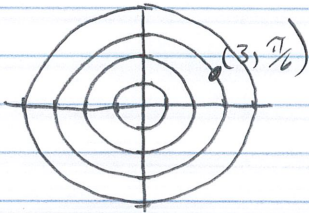
$$\theta \approx 23.41^\circ$$

$$4.36(\cos 23.41^\circ + i \sin 23.41^\circ)$$

$$\underline{\underline{\text{or } 4.36 \text{ cis } 23.41^\circ}}$$

#2) Graph on a polar grid and write in rectangular form: $z = 3(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$

(r, θ)
 $(3, \frac{\pi}{6})$



Rectangular Form!

$$3(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$$

$$3\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$\frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

$$\underline{\underline{2.60 + 1.5i}}$$

cw/hw: WS 8.5