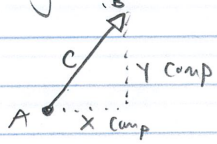
13-3 Vector Component Form, Magnitude, & Direction

Component Form: $\vec{v} = \langle x \text{ comp}, y \text{ comp} \rangle$

Given 2 points: (Initial) (x_1, y_1) and (Terminal) (x_2, y_2), then
component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$ (order matters - vectors have direction!)

Magnitude (distance formula): $\|v\| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ ← long way



$(x \text{ comp})^2 + (y \text{ comp})^2 = c^2$

$\sqrt{x^2+y^2} = \sqrt{c^2}$

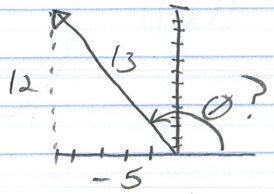
$c = \sqrt{x^2+y^2}$ ← bit of a shortcut

Ex 1) Ex: Find the component form and magnitude of $\overrightarrow{AB}$

$A = (4, -7)$ initial } Hold off at 1st
$B = (-1, 5)$ terminal }

Comp Form $= \langle -1-4, 5-(-7) \rangle = \langle -5, 12 \rangle$

Magnitude $= \sqrt{(-5)^2 + (12)^2}$
$= \sqrt{25+144}$
$= \sqrt{169}$
$= 13$

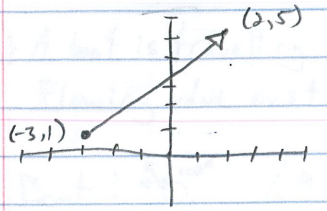


Given component Form $\langle a, b \rangle$: ↓ we use tan so we don't have to find a hypotenuse or use a rounded one

1) If $a > 0$, then $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ (Quad I/IV - the defaults)
2) If $a < 0$, then $\theta = \tan^{-1}\left(\frac{b}{a}\right) + 180°$ (Quad II/III, so we have to adjust)
3) If $a = 0$, then $\theta = 90°$ or $-90°$ ↑90° or ↓-90° (not 270°)

(cont)

Ex 2) Find the component form, magnitude, & direction



component form: $\langle 5, 4 \rangle$

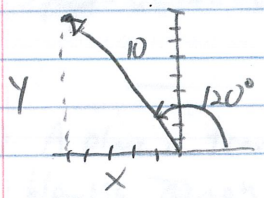
magnitude: $\sqrt{5^2+4^2} = \sqrt{41} \approx 6.40$

direction: $\tan^{-1}\left(\frac{4}{5}\right) \approx 38.66°$

↑ why no +180°?

Ex 3) Find the component form given the magnitude & direction:

magnitude = 10 $\quad \theta = 120°$



$\cos 120 = \frac{x}{10}$ $\qquad$ $\sin 120 = \frac{y}{10}$

$10 \cos 120 = x$ $\qquad$ $10 \sin 120 = y$

$-5 = x$ $\qquad$ $8.66 \approx y$

→ Short way: $\langle \|v\| \cos\theta, \|v\| \sin\theta \rangle$

$\langle 10 \cos 120, 10 \sin 120 \rangle$

$\langle -5, 8.66 \rangle$

$\|v\|$ = hypotenuse length every time

You try: Magnitude = 7.5 $\quad \theta = 78°$

$\langle \|v\| \cos\theta, \|v\| \sin\theta \rangle$

$\langle 7.5 \cos 78, 7.5 \sin 78 \rangle$

$\langle 1.56, 7.34 \rangle$ - reasonable?

CW/HW - wksht > 3