GSE PreCalculus: Unit 6: Law of Cosines/Law of Sines
Test 6 Review

Name Key

Solve the following oblique triangles using law of sines or cosines. Find all missing angle measures and side measures to the nearest tenth.

1. a= 27, b= 35, $\angle C = 71°$

A = 44.24° a = 27
B = 64.76° b = 35
C = 71° c = 36.59

A = _____ a = 27
B = _____ b = 35
C = 71 c = _____

$c^2 = 27^2 + 35^2 - 2(27)(35)\cos 71$

$c^2 = 1338.68$

$c = 36.59$

$27^2 = 35^2 + 36.59^2 - 2(35)(36.59)\cos A$

$-729 = 2563.83 - 2561.3\cos$

$\frac{-1834.83}{-2561.3} = \cos A$ $A = 44.24$

2. a = 5, b = 4, c = 7

A = 44.42° a = 5
B = 34.05° b = 4
C = 101.53° c = 7

A = _____ a = 5
B = _____ b = 4
C = _____ c = 7

$5^2 = 4^2 + 7^2 - 2(4)(7)\cos A$

$25 = 65 - 56\cos A$

$-40 = -56\cos A$

$\frac{-40}{-56} = \cos A$

$A = 44.42$

$4^2 = 5^2 + 7^2 - 2(5)(7)\cos A$

$16 = 74 - 70\cos A$

$-58 = -70\cos A$

$\frac{-58}{-70} = \cos A$

$A = 34.05$

3. $\angle B = 130°$, b = 5.2, c = 10.1

A = _____ a = _____
B = 130° b = 5.2 ← Small
C = DNE c = 10.1 ← Big

A = _____ a = _____
B = 130 b = 5.2
C = _____ c = 10.1

$\frac{\sin 130}{5.2} = \frac{\sin C}{10.1}$

$5.2\sin C = 10.1\sin 130$

$\sin C = \frac{10.1\sin 130}{5.2}$

C = Error

4. $\angle A = 73°$, b= 12.8, a= 12.5

A = 73° a = 12.5
B = 78.31° b = 12.8
C = 28.69° c = 6.28

A = 73 a = 12.5
B = 101.69° b = 12.8
C = 5.31° c = 1.21 or 1.46 w/LoC

$\frac{\sin 73}{12.5} = \frac{\sin B}{12.8}$

$12.5\sin B = 12.8\sin 73$

$\sin B = \frac{12.8\sin 73}{12.5}$

$B = \sin^{-1}\left(\frac{12.8\sin 73}{12.5}\right)$

$B = 78.31$

$\frac{\sin 73}{12.5} = \frac{\sin 28.69}{c}$

$\frac{c\sin 73}{\sin 73} = \frac{12.5\sin 28.69}{\sin 73}$

$c = 6.28$

$\frac{\sin 73}{12.5} = \frac{\sin 5.31}{c}$

$\frac{c\sin 73}{\sin 73} = \frac{12.5\sin 5.31}{\sin 73}$

c =

5. $\angle A = 150°$, b= 10, a= 64

A = 150° a = 64
B = 4.48° b = 10
C = 25.52° c = 55.15

A = 150 a = 64
B = 175.52 b = 10
C = -142.52 c = _____
DNE

$\frac{\sin 150}{64} = \frac{\sin B}{10}$

$64\sin B = 10\sin 150$

$\sin B = \frac{10\sin 150}{64}$

$B = \sin^{-1}\left(\frac{10\sin 150}{64}\right)$

$B = 4.48°$

$\frac{\sin 150}{64} = \frac{\sin 25.52}{c}$

$\frac{c\sin 150}{\sin 150} = \frac{64\sin 25.52}{\sin 150}$

$c = 55.15$

6. $\angle A = 27°18'$, b= 32.9, a= 27.4

A = 27°18' a = 27.4
B = 33.42° b = 32.9
C = 119.28° c = 52.11

A = 27°18' a = 27.4
B = 146.58 b = 32.9
C = 6.12 c = 6.37

$27°18' = 27.3°$

$\frac{\sin 27.3}{27.4} = \frac{\sin B}{32.9}$

$27.4\sin B = 32.9\sin 27.3$

$\sin B = \frac{32.9\sin 27.3}{27.4}$

$B = 33.42$

$\frac{\sin 27.3}{27.4} = \frac{\sin 119.28}{c}$

$\frac{c\sin 27.3}{\sin 27.3} = \frac{27.4\sin 119.28}{\sin 27.3}$

$c = 52.11$

$\frac{\sin 27.3}{27.4} = \frac{\sin 6.12}{c}$

$\frac{c\sin 27.3}{\sin 27.3} = \frac{27.4\sin 6.12}{\sin 27.3}$

$c = 6.37$

Find the area of the triangle having the indicated sides and angles. Round to the nearest tenth.

7. b= 22, a= 32, $\angle C = 128°$ $A_\Delta = \frac{1}{2}(32)(22)\sin 128°$ $277.4\ u^2$

$A_\Delta = 277.4$

8. b= 18, c =22 , $\angle C = 128°$ ~~$255.2\ u^2$~~ $40.64\ u^2$ or $40.75\ u^2$

~~$c^2 = 18^2 + 22^2 - 2(18)(22)\cos 128$~~ ~~$A_\Delta = \frac{1}{2}(39.91)(18)\sin 128$~~

~~$c^2 = 1045$~~ ~~$\alpha = 35.09$~~ ~~$A_\Delta = 255.2$~~ See attached

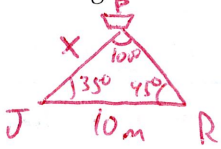
9. A triangular parking lot has sides of 150 feet, 210 feet, and 190 feet. Find the area of the triangle.

$s = \frac{1}{2}(150+210+190)$ $A_\Delta = \sqrt{275(275-150)(275-210)(275-190)}$ $13781.2\ u^2$

$s = 275$ $A_\Delta =$

Draw a picture and solve.

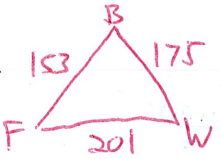
10. Juan and Rebekah are standing at the seashore 10 miles apart. The coastline is a straight line between them. Both can see the same ship in the water. The angle between the coastline and the line between the ship and Juan is 35 degrees. The angle between the coastline and the line between the ship and Romelia is 45 degrees. How far is the ship from Juan?



$\frac{\sin 100}{10} = \frac{\sin 45}{x}$ $x = \frac{10\sin 45}{\sin 100}$

$x \sin 100 = 10 \sin 45$ $x = 7.18$ miles

11. Fred, Barney and Wilma are camping in their tents. If the distance between Fred and Barney is 153 feet, the distance between Fred and Wilma is 201 feet, and the distance between Barney and Wilma is 175 feet, find each angle of the triangle formed.



$201^2 = 175^2 + 153^2 - 2(175)(153)\cos B$ $175^2 = 201^2 + 153^2 - 2(201)(153)\cos F$

$40401 = 54034 - 53550\cos B$ $30625 = 63810 - 61506\cos F$

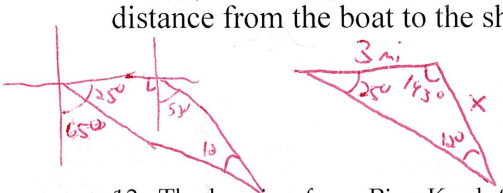
$-13633 = -53550\cos B$ $-33185 = -61506\cos F$

$B = 75.25°$ $F = 57.35°$

180
- 75.25
- 57.35

$W = 47.40°$

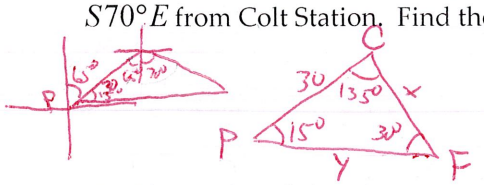
12. A boat is sailing due east parallel to the given shoreline at a speed of 12 mph. At a given time the bearing to the lighthouse is $S65°E$, and 15 minutes later the bearing is $S53°E$. Find the distance from the boat to the shoreline if the lighthouse is on the shoreline.



$\frac{\sin 12}{3} = \frac{\sin 25}{x}$ $x = 6.10$ mi

$\frac{x\sin 12}{\sin 12} = \frac{3\sin 25}{\sin 12}$

13. The bearing from Pine Knob fire tower to the Colt Station fire tower is $N65°E$ and the two towers are 30 km apart. A fire spotted by rangers in each tower has a bearing of $N80°E$ from Pine Knob and $S70°E$ from Colt Station. Find the distance of the fire from each tower.



$\frac{\sin 30}{30} = \frac{\sin 15}{x}$ $\frac{\sin 30}{30} = \frac{\sin 135}{y}$

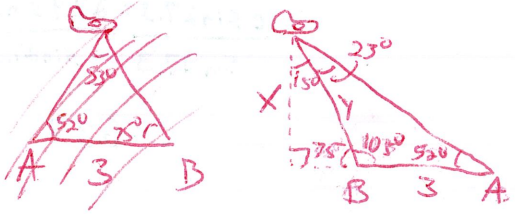
$\frac{x\sin 30}{\sin 30} = \frac{30\sin 15}{\sin 30}$ $\frac{y\sin 30}{\sin 30} = \frac{30\sin 135}{\sin 30}$

$x = 15.53$ $y = 42.43$

Pine knob = 42.43 km

Colt = 15.53 km

14. The angles of elevation to an airplane from two points A and B on level ground is 52° and 75°, respectively. The points A and B are 3 miles apart , and the airplane is west of both points in the same vertical plane. Find the altitude of the plane.



$\frac{\sin 23°}{3} = \frac{\sin 52°}{y}$ $\sin 75° = \frac{x}{6.05}$

$\frac{y\sin 23}{\sin 23} = \frac{3\sin 52}{\sin 23}$ $6.05\sin 75 = x$

$y = 6.05$ mi $x = 5.84$ miles

Review from Test 1-Test 5B:

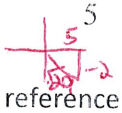
1. Identify the following conics: a. $\frac{(x-3)^2}{25}+\frac{y^2}{9}=1$ Ellipse b. $(x+1)^2-\frac{y^2}{25}=16$ Hyperbola

2. Multiply the following matrices: $\begin{bmatrix}4 & y\\ 2 & x\end{bmatrix}\cdot\begin{bmatrix}3 & 2\\ -2 & 1\end{bmatrix}$ $\begin{bmatrix}12-2y & 8+y\\ 6-2x & 4+x\end{bmatrix}$

3. Solve the linear system: $\begin{aligned}2x-5y&=8\\ x+2y&=12\end{aligned}$ $\begin{bmatrix}2 & -5\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}8\\ 12\end{bmatrix}$ $x=\frac{76}{9}$, $y=\frac{16}{9}$ $\left(\frac{76}{9},\frac{16}{9}\right)$

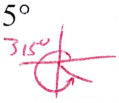
4. Find a positive co-terminal angle to: a. $\theta=-\frac{\pi}{3}=\frac{5\pi}{3}$ b. $\theta=\frac{3\pi}{5}=\frac{13\pi}{5}$

5. . If $\tan\theta=-\frac{2}{5}$ and θ is in quadrant 4, what is the exact value of sin θ?



$=\frac{-2}{\sqrt{29}}=\frac{-2\sqrt{29}}{29}$

6. Find the reference angle: a. $\theta=120°$ 60° b. $\theta=315°$ 45°



7. Find the exact value of the following function: $\sin\left(-\frac{4\pi}{3}\right)$ $\frac{\sqrt{3}}{2}$

8. Evaluate $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ in degrees and radians

30°, π/6

9. Find the amplitude, period, horizontal shift, and vertical shift for $f(x)=3\sin\left(x+\frac{\pi}{4}\right)$.

Amplitude = 3 Period = $2\pi/1=2\pi$ Horiz. shift = $-\pi/4$ Vert. shift = 0

10. Show that $\cos x-\cos x\sin^2 x=\cos^3 x$

$\cos x(1-\sin^2 x)=\cos^3 x$

$\cos x(\cos^2 x)=\cos^3 x$

$\cos^3 x=\cos^3 x$ ✓

11. Solve for x: $2\sin x-1=0$

$2\sin x=1$

$\sin x=\frac{1}{2}$

$x=\sin^{-1}\left(\frac{1}{2}\right)$

$x=30°$

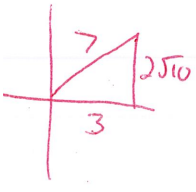
12. Evaluate: cos105° (Use the fact that 105° = 60° + 45°.)

Use $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$\cos60(\cos45)-\sin60(\sin45)$

$\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}=\frac{\sqrt{2}-\sqrt{6}}{4}$

13. Given $\cos u=\frac{3}{7}$ and u is in quadrant 1, find sin 2u. (Use: sin (2α) = 2 sinα cosα)



$\sin 2u=2\sin u\cos u$

$=\frac{2}{1}\left(\frac{2\sqrt{10}}{7}\right)\left(\frac{3}{7}\right)$

$=\frac{12\sqrt{10}}{49}$

#8 $A = 11.85°$ $a = 5.73$

$B = 40.15°$ $b = 18$

$C = 128°$ $c = 22$

$\frac{\sin B}{18} = \frac{\sin 128}{22}$

$\frac{22 \sin B}{22} = \frac{18 \sin 128}{22}$

$\sin B = \frac{18 \sin 128}{22}$

$B = 40.15°$

$A = -87.85$

$B = 139.85$

$C = 128$

DNE

$\frac{\sin 11.85}{a} = \frac{\sin 128}{22}$

$\frac{a \sin 128}{\sin 128} = \frac{22 \sin 11.85}{\sin 128}$

$a = 5.73$

Option 1) $A_\Delta = \frac{1}{2}(5.73)(18) \sin 128$

$A_\Delta = 40.64 \text{ u}^2$

Option 2) $s = \frac{1}{2}(5.73 + 18 + 22) = 22.87$

$A_\Delta = \sqrt{22.87(22.87-5.73)(22.87-18)(22.87-22)}$

$A_\Delta = 40.75$