

**Two ways to find the Area of a triangle:**

1.  $A_s = \frac{1}{2}ab \sin C$       **Given 2 sides and 1 angle – all letters different; a, b = sides, C = angle**

Example: Find the area:  $p = 6.8$  in,  $k = 16$  in,  $H = 111^\circ$

$$A = \frac{1}{2}kp \sin H$$

$$A = \frac{1}{2}(16)(6.8) \sin 111^\circ$$

$$A \approx 50.79 \text{ in}^2$$

**2. Heron's Area Formula - Given 3 sides of the triangle**

The area of a triangle with sides of length  $a$ ,  $b$ , and  $c$  is

$$A_s = \sqrt{s(s-a)(s-b)(s-c)}$$

Where  $s = \frac{1}{2}(a+b+c)$ . The variable  $s$  is called the *semiperimeter*, or half-perimeter, of the triangle

**Example:** Find the area of the triangle to the right (nearest tenth).

**Step 1:** Find the semiperimeter:

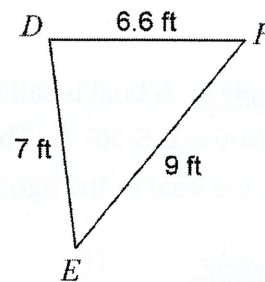
$$s = \frac{1}{2}(7 + 6.6 + 9) = 11.3$$

**Step 2:** Use Heron's formula:

$$A = \sqrt{11.3(11.3-7)(11.3-6.6)(11.3-9)}$$

$$A = \sqrt{525.2579}$$

$$A \approx 22.92$$



**Exercises:** Find the area of the triangle to the nearest tenth (show work).

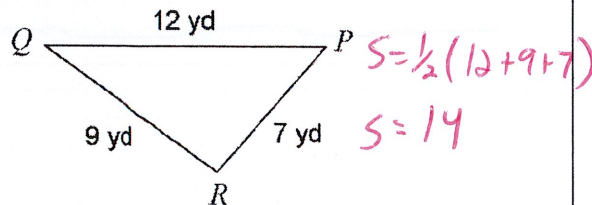
1.  $x = 14, y = 28, Z = 87^\circ$

$$A = \frac{1}{2}xy \sin Z$$

$$A = \frac{1}{2}(14)(28) \sin 87$$

$$A = 195.73$$

2.



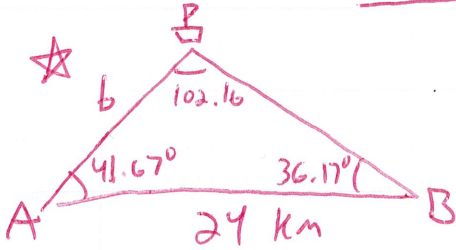
$$s = \frac{1}{2}(12 + 9 + 7)$$

$$s = 14$$

$$A = \sqrt{14(14-12)(14-9)(14-7)} \approx 31.30$$

Draw a picture and solve.

**Example 1:** A boat in distress at sea is sighted from two coast guard observation posts, A and B, on the shore. The angle at post A formed by lines of sight to post B and the boat is  $41.67^\circ$ . The angle at post B formed by the lines of sight to post A and the boat is  $36.17^\circ$ . The distance from Post A to Post B is 24 km. Find the distance from Post A to the boat.

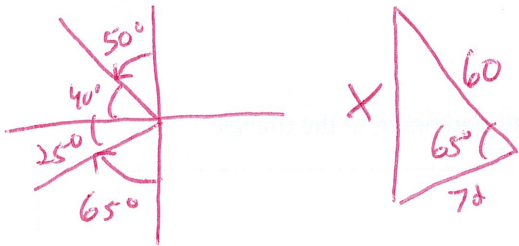


$$\frac{\sin 102.16}{24} = \frac{\sin 36.17}{b}$$

$$b \sin 102.16 = \frac{24 \sin 36.17}{\sin 102.16}$$

$$b = 14.49 \text{ km}$$

**Example 2:** Two ships leave a port at 8:00AM. One travels at a bearing of  $N50^\circ W$  at 15 mph and the other travels at a bearing of  $S65^\circ W$  at 18 mph. Approximate how far apart they are at noon that day.

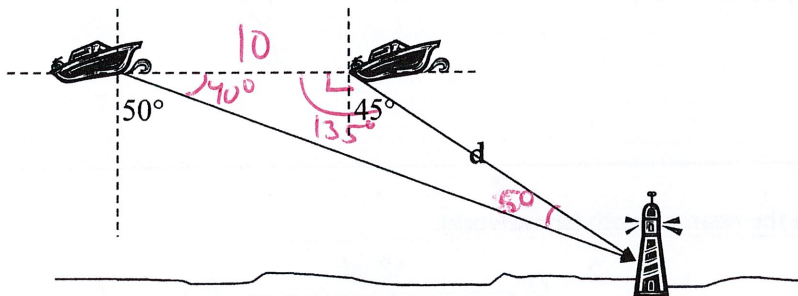


$$x^2 = 60^2 + 72^2 - 2(60)(72)\cos 65^\circ$$

$$x^2 = 5132.58$$

$$x = 71.64 \text{ miles}$$

**Example 3:** A boat is sailing due east parallel to the shoreline. At a given time the bearing to the lighthouse is  $S 50^\circ E$ . The boat travels 10 miles and the bearing is now  $S 45^\circ E$ . Find the distance from the boat to the lighthouse (d)



$$\frac{\sin 50}{10} = \frac{\sin 40}{d}$$

$$\frac{d \sin 50}{\sin 5} = \frac{10 \sin 40}{\sin 5}$$

$$d = 73.75 \text{ miles}$$