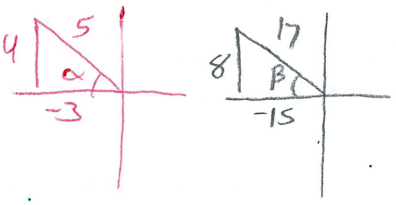
GSE PreCalculus **Name** Key

Test 5B Review: Trig Identities

Know the following:

1. Unit Circle – finding the exact value of angle (i.e. cos 60)
2. Inverses
   a. Identify the graphs of inverse functions (sin, cos, tan)
   b. How to find the inverse given the coordinate (i.e. $\cos^{-1}\left(\frac{1}{2}\right)$ - answer in degrees and radians)

Given that α and β are in quadrant 2 and $\sin\alpha = \frac{4}{5}$ and $\cos\beta = -\frac{15}{17}$, find:



1. $\cos\alpha$ $-3/5$

2. $\sin\beta$ $8/17$

3. $\sin(2\alpha)$ $2\sin\alpha\cos\alpha = 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right) = \boxed{\frac{-24}{25}}$

4. $\cos(2\beta)$ $\cos^2\beta - \sin^2\beta = \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2 = \frac{225}{289} - \frac{64}{289} = \boxed{\frac{161}{289}}$

5. $\tan(2\beta)$ $\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\left(\frac{8}{15}\right)}{1-\left(\frac{8}{15}\right)^2} = \frac{\frac{16}{15}}{1-\frac{64}{225}} = \frac{\frac{16}{15}}{\frac{161}{225}} = \frac{16}{-15}\cdot\frac{225}{161} = \boxed{\frac{-240}{161}}$

6. $\cos(\alpha+\beta)$ $\cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{-3}{5}\left(\frac{-15}{17}\right) - \left(\frac{4}{5}\right)\left(\frac{8}{17}\right) = \frac{45}{85} - \frac{32}{85} = \boxed{\frac{13}{85}}$

Use half angle formulas to solve the following

7. $\cos 157.5° = \sqrt{\frac{1+\cos 315}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$
   ↓
   $315/2$

8. $\sin 15° = \sqrt{\frac{1-\cos 30}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$
   ↓
   $30/2$

Solve.

9. $2\sin^2 x = 2 + \cos x$
   $2(1-\cos^2 x) = 2+\cos x$

10. $2\sin\alpha\cos\alpha = \sin\alpha$

11. $\sin^2 x - 3\cos x = 3$

Attached

12. $2\sin^2 x = 9\sin x + 5$

13. $\sin^2\beta - \sin\beta = 0$

Verify the following.

14. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

15. $\dfrac{\sin x}{\sin x-\cos x}=\dfrac{1}{1-\cot x}$

16. $\sec^4 x-\tan^4 x=1+2\tan^2 x$

16. $\cos^2 x(1+\tan^2 x)=1$

17. $\csc 2\theta=\dfrac{\csc\theta}{2\cos\theta}$

18. $\sec 2\theta=\dfrac{\sec^2\theta}{2-\sec^2\theta}$

19. $\cos^4 x-\sin^4 x=\cos 2x$

20. $(\sin x+\cos x)^2=1+\sin 2x$

Cumulative Review from Test 1-5A:

21. Identify the following conics: a. $\dfrac{(x-3)^2}{25}+\dfrac{y^2}{9}=1$ Ellipse

b. $(x+1)^2+y^2=16$ Circle

22. Multiply the following matrices: $\begin{bmatrix} x & -1 \\ 2 & 3 \end{bmatrix}\cdot\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$ $\begin{bmatrix} 3x+2 & 2x-1 \\ 0 & 7 \end{bmatrix}$

23. Solve the linear system: $\begin{aligned} 2x+4y&=8 \\ x-2y&=12 \end{aligned}$ $\begin{bmatrix} 2 & 4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ 12 \end{bmatrix}$ $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ -2 \end{bmatrix}$ $x=8,\ y=-2$

24. Find a positive co-terminal angle to: a. $\theta=-\dfrac{2\pi}{7}$ $\dfrac{12\pi}{7}$ b. $\theta=\dfrac{\pi}{5}$ $\dfrac{11\pi}{5}$

25. . If $\tan\theta=\dfrac{5}{12}$ and θ is in quadrant 3, what is the exact value of cos θ? $\cos\theta=\dfrac{-12}{13}$

26. Find the reference angle: a. $\theta=120°$ $60°$ b. $\theta=315°$ $45°$

27. Find the exact value of the following function: $\sin\left(-\dfrac{4\pi}{3}\right)$ $\dfrac{\sqrt{3}}{2}$

28. Evaluate: $\cos^{-1}\left(\dfrac{1}{2}\right)$ $60°$ or $\dfrac{\pi}{3}$

9) $2\sin^2 x = 2 + \cos x$

$2(1-\cos^2 x) = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$-2\cos^2 x - \cos x = 0$

$\cos x\,(-2\cos x - 1) = 0$

$\cos x = 0$ | $-2\cos x - 1 = 0$

$x = 90$ or $\pi/2$ | $-2\cos x = 1$

$\cos x = -1/2$

$270°$ | $x = 120°$ or $2\pi/3$

~~240°~~

10) $2\sin\alpha\cos\alpha = \sin\alpha$

$\sin 2\alpha = \sin\alpha$

$\sin^{-1}(\sin 2\alpha) = \sin^{-1}(\sin\alpha)$

$2\alpha = \alpha$

$-\alpha \quad -\alpha$

$\alpha = 0$

11) $\sin^2 x - 3\cos x = 3$

$1 - \cos^2 x - 3\cos x = 3$

$-\cos^2 x - 3\cos x - 2 = 0$

$\cos^2 x + 3\cos x + 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$

$\cos x + 1 = 0$ | $\cos x + 2 = 0$

$\cos x = -1$ | $\cos x = -2$

$x = 180°$ or π | Nope!

12) $2\sin^2 x = 9\sin x + 5$

$2\sin^2 x - 9\sin x - 5 = 0$

$(2\sin x + 1)(\sin x - 5) = 0$

$2\sin x + 1 = 0$ | $\sin x - 5 = 0$

$2\sin x = -1$ | $\sin x = 5$

$\sin x = -1/2$ | Nope!

$x = 330°$ or $11\pi/6$

13) $\sin^2\beta - \sin\beta = 0$

$\sin\beta(\sin\beta - 1) = 0$

$\sin\beta = 0$ | $\sin\beta - 1 = 0$

$\beta = 0°$ | $\sin\beta = 1$

or 0 | $\beta = 90°$ or $\pi/2$

10) $2\sin\alpha\cos\alpha = \sin\alpha$

$2\sin\alpha\cos\alpha - \sin\alpha = 0$

$\sin\alpha\,(2\cos\alpha - 1) = 0$

$\sin\alpha = 0$ | $2\cos\alpha - 1 = 0$

$\alpha = 0$ | $2\cos\alpha = 1$

$\cos\alpha = 1/2$

$\alpha = 60°$

14) $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$\sin x\cos y+\cos x\sin y+\sin x\cos y-\cos x\sin y=2\sin x\cos x$

$2\sin x\cos y=2\sin x\cos y$ ✓

15) $\frac{\sin x}{\sin x-\cos x}=\frac{1}{1-\cot x}$

$\frac{\sin x}{\sin x\left(1-\frac{\cos x}{\sin x}\right)}=\frac{1}{1-\cot x}$

$\frac{1}{1-\cot x}=\frac{1}{1-\cot x}$ ✓

16) $\sec^4 x-\tan^4 x=1+2\tan^2 x$

$(\sec^2 x+\tan^2 x)(\sec^2 x-\tan^2 x)=1+2\tan^2 x$

$(1+\tan^2 x+\tan^2 x)(1+\tan^2 x-\tan^2 x)=1+2\tan^2 x$

$(1+2\tan^2 x)(1)=1+2\tan^2 x$

$1+2\tan^2 x=1+2\tan^2 x$ ✓

16 pt2) $\cos^2 x(1+\tan^2 x)=1$

$\cos^2 x(\sec^2 x)=1$

$\frac{\cos^2 x}{1}\cdot\left(\frac{1}{\cos^2 x}\right)=1$

$1=1$ ✓

17) $\csc 2\theta=\frac{\csc\theta}{2\cos\theta}$

$\frac{1}{\sin 2\theta}=\frac{\csc\theta}{2\cos\theta}$

$\frac{1}{2\sin\theta\cos\theta}=\frac{\csc\theta}{2\cos\theta}$

$\frac{1}{\sin\theta}\cdot\frac{1}{2\cos\theta}=\frac{\csc\theta}{2\cos\theta}$

$\frac{\csc\theta}{1}\cdot\frac{1}{2\cos\theta}=\frac{\csc\theta}{2\cos\theta}$

$\frac{\csc\theta}{2\cos\theta}=\frac{\csc\theta}{2\cos\theta}$ ✓

18) $\sec 2\theta=\frac{\sec^2\theta}{2-\sec^2\theta}$

$\frac{1}{\cos 2\theta}=\frac{\sec^2\theta}{2-\sec^2\theta}$

$\frac{1}{2\cos^2\theta-1}=\frac{\sec^2\theta}{2-\sec^2\theta}$

$\frac{1/\cos^2\theta}{1/\cos^2\theta}\cdot\frac{1}{2\cos^2\theta-1}=\frac{\sec^2\theta}{2-\sec^2\theta}$

$\frac{1/\cos^2\theta}{2-(1/\cos^2\theta)}=\frac{\sec^2\theta}{2-\sec^2\theta}$

$\frac{\sec^2\theta}{2-\sec^2\theta}=\frac{\sec^2\theta}{2-\sec^2\theta}$ ✓

19) $\cos^4 x-\sin^4 x=\cos 2x$

$(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)=\cos 2x$

$1(\cos^2 x-\sin^2 x)=\cos 2x$

$\cos^2 x-\sin^2 x=\cos 2x$

$\cos 2x=\cos 2x$ ✓

20) $(\sin x+\cos x)^2=1+\sin 2x$

$\sin^2 x+2\sin x\cos x+\cos^2 x=1+\sin 2x$

$\sin^2 x+\cos^2 x+2\sin x\cos x=1+\sin 2x$

$1+\sin 2x=1+\sin 2x$ ✓