

10-4

Solving Trig Equations w/o a Calculator

Finding Inverses (Reminder) - We're going past this, though

$$\sin^{-1}(x) / \arcsin x \rightarrow \text{Quad I \& IV}$$

$$\cos^{-1}(x) / \arccos x \rightarrow \text{Quad I \& II}$$

$$\tan^{-1}(x) / \arctan x \rightarrow \text{Quad I \& IV}$$

Ex 1: Verify that the x-values are solutions

$$2 \sin^2 x + 3 \sin x + 1 = 0$$

$$\begin{array}{l} \text{a) } \frac{\pi}{6} \times \\ \text{b) } 210^\circ \checkmark \end{array}$$

$$\text{a) } 2 \sin^2 \frac{\pi}{6} + 3 \sin \frac{\pi}{6} + 1 = 0$$

$$2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1 = 0$$

All positive. Nope.

$$\text{b) } 2 \sin^2 210^\circ + 3 \sin 210^\circ + 1 = 0$$

$$2\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1 = 0$$

$$2\left(\frac{1}{4}\right) - \frac{3}{2} + 1 = 0$$

$$\frac{1}{2} - \frac{3}{2} + \frac{2}{2} = 0 \checkmark$$

Ex 2: Solve $2 \sin x - 1 = 0$

$0^\circ \leq x \leq 360^\circ$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$\sin^{-1}(\sin x) = \sin^{-1} \frac{1}{2}$$

$$x = 30^\circ, 150^\circ$$

Ex 3: Solve $3 \tan^2 x - 1 = 0$

$0^\circ \leq x \leq 360^\circ$

$$3 \tan^2 x = 1$$

$$\tan^2 x = \frac{1}{3} \star$$

$$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1}{\sqrt{3}}$$

$$\tan^{-1}(\tan x) = \tan^{-1} \frac{1}{\sqrt{3}}$$

$$x = 30^\circ$$

$$210^\circ$$

$$\text{and } \tan^{-1}(\tan x) = \tan^{-1} \frac{-1}{\sqrt{3}}$$

$$x = 330^\circ$$

$$150^\circ$$

(cont)

Ex 4: Solve $\cot x \cos^2 x = 2 \cot x$ Default Quadrants

$\cot x \cos^2 x - 2 \cot x = 0$ Radians

$\cot x (\cos^2 x - 2) = 0$

$\cot x = 0$

$\cos^2 x - 2 = 0$

$\frac{\cos x}{\sin x} = \frac{0}{1}, \frac{0}{-1}$

$\cos^2 x = 2$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = \pm \sqrt{2}$

Impossible $\sqrt{2} > 1$

Ex 5: $2 \sin^2 x - \sin x - 1 = 0$ $0^\circ \leq x < 360^\circ$

$(2 \sin x + 1)(\sin x - 1) = 0$

$2 \sin x + 1 = 0$

$\sin x - 1 = 0$

$2 \sin x = -1$

$\sin x = 1$

$\sin x = -\frac{1}{2}$

$x = 90^\circ$

$x = 330^\circ,$

210°

$x = 90^\circ, 210^\circ, 330^\circ$

Ex 6: $\csc^2 x - 2 = 0$

$0 \leq x \leq \pi$

$\csc^2 x = 2$

$\csc x = \pm \sqrt{2}$

$\frac{1}{\sin x} = \pm \sqrt{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

CW/HW - worksheet