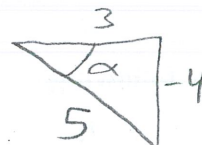
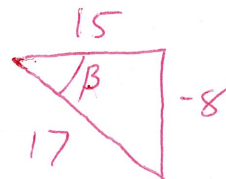


Given that α and β are in quadrant 4 and $\sin \alpha = -\frac{4}{5}$ and $\cos \beta = \frac{15}{17}$, find:



1. $\cos(\alpha) = \frac{3}{5}$

2. $\sin(\beta) = \frac{-8}{17}$

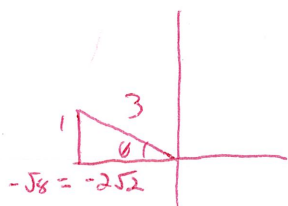


3. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = -\frac{4}{5} \left(\frac{15}{17}\right) + \frac{3}{5} \left(\frac{-8}{17}\right) = \frac{-60}{85} + \frac{-24}{85} = \frac{-84}{85}$

4. $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = \frac{3}{5} \left(\frac{15}{17}\right) + \left(-\frac{4}{5}\right) \left(\frac{-8}{17}\right) = \frac{45}{85} + \frac{32}{85} = \frac{77}{85}$

5. $\cot(\alpha - \beta) = \frac{\cos(\alpha - \beta)}{\sin(\alpha - \beta)} = \frac{77/85}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \frac{77/85}{-\frac{4}{5} \left(\frac{15}{17}\right) - \frac{3}{5} \left(\frac{-8}{17}\right)} = \frac{77/85}{\frac{-60}{85} - \frac{24}{85}} = \frac{77/85}{-36/85} = \frac{77}{-36}$

6. If $\sin \theta = \frac{1}{3}$ and $90^\circ < \theta < 180^\circ$, then find the value of $\sec \theta$



$\cos \theta = \frac{-2\sqrt{2}}{3}$

$\sec \theta = \frac{3}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-3\sqrt{2}}{4}$

Use sum/difference formulas to find the exact value of the following:

1. $\sin 60^\circ = \sin(90^\circ - 30^\circ) = \sin 90^\circ \cos 30^\circ - \cos 90^\circ \sin 30^\circ = 1 \left(\frac{\sqrt{3}}{2}\right) - 0 \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

2. $\cos 75^\circ = \cos(120^\circ - 45^\circ) = \cos 120^\circ \cos 45^\circ + \sin 120^\circ \sin 45^\circ = \left(-\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{\sqrt{6} - \sqrt{2}}{4}$

Write as the sin, cos, or tan of a single angle.

1. $\sin 70^\circ \cos 40^\circ - \cos 70^\circ \sin 40^\circ = \sin(70 - 40) = \sin 30^\circ$

2. $\cos 210^\circ \cos 80^\circ + \sin 210^\circ \sin 80^\circ = \cos(210 - 80) = \cos 130^\circ$

Verify the following.

1. $\sec \theta \cot \theta = \csc \theta$

$$\frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \csc \theta$$

$$\frac{1}{\sin \theta} = \csc \theta \quad \csc \theta = \csc \theta \checkmark$$

3. $\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$

$$\sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y = 2 \sin x \cos y$$

$$\sin x \cos y + \sin x \cos y = 2 \sin x \cos y$$

$$2 \sin x \cos y = 2 \sin x \cos y$$

5. $\frac{\sec^2 \theta}{\tan \theta} = \sec \theta \csc \theta$

$$\frac{\sqrt{\cos^2 \theta}}{\sin \theta / \cos \theta} = \sec \theta \csc \theta$$

$$\frac{\cos \theta \sin \theta}{\cos^2 \theta \sin \theta} = \sec \theta \csc \theta$$

$$\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \sec \theta \csc \theta \quad \sec \theta \csc \theta = \sec \theta \csc \theta \checkmark$$

7. $1 + \sec^2 \theta \sin^2 \theta = \sec^2 \theta$

$$1 + \left(\frac{1}{\cos^2 \theta}\right) \left(\frac{\sin^2 \theta}{1}\right) = \sec^2 \theta$$

$$1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \sec^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sec^2 \theta = \sec^2 \theta \checkmark$$

9. $\frac{\sin^2 \theta + 5 \sin \theta + 6}{\sin^2 \theta - 4} = \frac{\sin \theta + 3}{\sin \theta - 2}$

$$\frac{(\sin \theta + 2)(\sin \theta + 3)}{(\sin \theta + 2)(\sin \theta - 2)} = \frac{\sin \theta + 3}{\sin \theta - 2}$$

$$\frac{\sin \theta + 3}{\sin \theta - 2} = \frac{\sin \theta + 3}{\sin \theta - 2} \checkmark$$

2. $\sin \theta \csc \theta - \sin^2 \theta = \cos^2 \theta$

$$\sin \theta (\frac{1}{\sin \theta}) - \sin^2 \theta = \cos^2 \theta$$

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$\cos^2 \theta = \cos^2 \theta \checkmark$$

4. $\frac{\csc \theta}{\sec \theta} + \frac{\cos \theta}{\sin \theta} = 2 \cot \theta$

$$\frac{1/\sin \theta}{1/\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \cot \theta$$

$$\frac{\cos \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = 2 \cot \theta$$

$$\cot \theta + \cot \theta = 2 \cot \theta$$

$$2 \cot \theta = 2 \cot \theta \checkmark$$

6. $\cos^2 x (1 + \tan^2 x) = 1$

$$\cos^2 x (\sec^2 x) = 1$$

$$\frac{\cos^2 x}{1} \cdot \frac{1}{\cos^2 x} = 1$$

$$1 = 1 \checkmark$$

8. $\frac{1}{1 - \cos x} - \frac{1}{1 + \cos x} = 2 \csc x \cot x$

$$\frac{1 + \cos x}{1 - \cos^2 x} - \frac{1 - \cos x}{1 - \cos^2 x} = 2 \csc x \cot x$$

$$\frac{2 \cos x}{1 - \cos^2 x} = 2 \csc x \cot x$$

$$\frac{2 \cos x}{\sin^2 x} = 2 \csc x \cot x$$

10. $\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = 2 \csc x \cot x \quad 2 \csc x \cot x = 2 \csc x \cot x$
 $\cos x (1 - 2 \cos^2 x + \cos^4 x) = \sin^5 x$

$$\sin x (1 - \cos^2 x)(1 - \cos^2 x) = \sin^5 x$$

$$\sin x (\sin^2 x)(\sin^2 x) = \sin^5 x$$

$$\sin^5 x = \sin^5 x \checkmark$$