GPS PreCalculus
Sum/Difference – Guided Notes

$\sin(a+b) = \sin a \cos b + \cos a \sin b$
$\sin(a-b) = \sin a \cos b - \cos a \sin b$
$\cos(a-b) = \cos a \cos b + \sin a \sin b$
$\cos(a+b) = \cos a \cos b - \sin a \sin b$
$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$
$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

Sum/Difference Identities

$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$ $\quad + \to +, \; - \to -$

$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$ $\quad - \to +, \; + \to -$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$ $\quad + \to \begin{smallmatrix}+\\-\end{smallmatrix}, \; - \to \begin{smallmatrix}-\\+\end{smallmatrix}$

Example 1: Use the sum and difference identities to find the exact value of sin/cos/tan

$15° = (45° - 30°)$

a. $\sin 15°$ $\quad \sin(45-30) = \sin 45 \cos 30 - \cos 45 \sin 30$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\boxed{= \frac{\sqrt{6} - \sqrt{2}}{4}}$

b. $\cos 15°$ $\quad \cos(45-30) = \cos 45 \cos 30 + \sin 45 \sin 30$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\boxed{= \frac{\sqrt{6} + \sqrt{2}}{4}}$

c. $\tan 15°$ $\quad \tan(45-30) = \dfrac{\tan 45 - \tan 30}{1 + \tan 45(\tan 30)}$ ← oh dear....

OR $\tan 15 = \dfrac{\sin 15}{\cos 15}$

$= \dfrac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \dfrac{\sqrt{6} - \sqrt{2}}{4} \cdot \dfrac{4}{\sqrt{6}+\sqrt{2}} = \boxed{\dfrac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}}$

Example 2: Rewrite the expression using sin, cos, or tan: $\sin 340^\circ \cos 50^\circ - \cos 340^\circ \sin 50^\circ$

$\alpha \quad \beta \quad \alpha \quad \beta$

$\sin(340 - 50)$

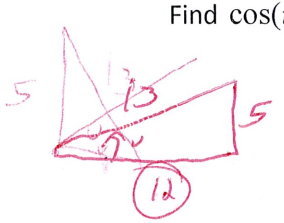
$\boxed{\sin 290^\circ}$

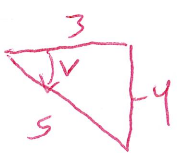
Example 3: Find the exact value of the trig function given:

$\sin u = \frac{5}{13} \qquad 0 < u < \frac{\pi}{2}$

$\cos v = \frac{3}{5} \qquad \frac{3\pi}{2} < v < 2\pi$

Find $\cos(u+v)$





$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$= \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{-4}{5}\right)$$

$$= \frac{36}{65} - \frac{-20}{65}$$

$$\boxed{= \frac{56}{65}}$$

Example 4: Verify $\sin(x+\frac{\pi}{3}) + \sin(x-\frac{\pi}{3}) = \cancel{1} \sin x$

$$\sin x \cos \tfrac{\pi}{3} + \cos x \sin \tfrac{\pi}{3} + \sin x \cos \tfrac{\pi}{3} - \cos x \sin \tfrac{\pi}{3} = \sin x$$

$$\sin x \left(\tfrac{1}{2}\right) + \cos x \left(\tfrac{\sqrt{3}}{2}\right) + \sin x \left(\tfrac{1}{2}\right) - \cos x \left(\tfrac{\sqrt{3}}{2}\right) = \sin x$$

$$\tfrac{1}{2} \sin x + \cancel{\tfrac{\sqrt{3}}{2} \cos x} + \tfrac{1}{2} \sin x - \cancel{\tfrac{\sqrt{3}}{2} \cos x} = \sin x$$

$$\sin x = \sin x \checkmark$$