

4-2

# Classifying Conics

Circle:  $(x-h)^2 + (y-k)^2 = r^2$

Parabola:  $(x-h)^2 = 4p(y-k)$  or  $(y-k)^2 = 4p(x-h)$

Ellipse:  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$  or  $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Hyperbola:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$  or  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

General Form for all conics:  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$   
Always a 0 for us

How to tell

Is  $x^2$  or  $y^2$  missing? Yes = Parabola  $6x^2 + 3x - 9y + 12 = 0$   
 No = keep going

Is  $x^2$  or  $y^2$  negative? Yes = Hyperbola  $4x^2 - 6y^2 + 7x - 3y - 18 = 0$   
 No = keep going

Does  $A=C$ ? Yes = Circle  $7x^2 + 7y^2 + 49x + 35y - 70 = 0$   
 No = Ellipse  $5x^2 + 10y^2 - 25x + 15y - 130 = 0$

- In general:
- 1) Classify them
  - 2) Put them in order
  - 3) Complete the square
  - 4) Maybe divide (if you want)

(cont)



Circle, but  
don't feel them  
right away

$$\begin{aligned}
 2x^2 + 2y^2 - 8x - 12y - 8 &= 0 \\
 x^2 + y^2 - 4x - 6y - 4 &= 0 \\
 x^2 - 4x + y^2 - 6y &= 4 \\
 (x^2 - 4x + 4) + (y^2 - 6y + 9) &= 4 + 4 + 9 \\
 (x-2)^2 + (y-3)^2 &= 17
 \end{aligned}$$

Ellipse, but  
still hold off

$$\begin{aligned}
 4x^2 + 25y^2 - 8x + 150y - 71 &= 0 \\
 4x^2 - 8x + 25y^2 + 150y &= 71 \\
 4(x^2 - 2x) + 25(y^2 + 6y) &= 71 \\
 4(x^2 - 2x + 1) + 25(y^2 + 6y + 9) &= 71 + 4 + 225 \\
 4(x-1)^2 + 25(y+3)^2 &= 300 \\
 \frac{4(x-1)^2}{4 \cdot 25} + \frac{25(y+3)^2}{25 \cdot 12} &= \frac{300}{300} \\
 \frac{(x-1)^2}{25} + \frac{(y+3)^2}{12} &= 1 \leftarrow \star
 \end{aligned}$$

Hyperbola

Why is  $\rightarrow$   
y 1st?

$$\begin{aligned}
 -15x^2 + 49y^2 - 30x - 98y - 701 &= 0 \\
 49y^2 - 98y - 15x^2 - 30x &= 701 \\
 49(y^2 - 2y) - 15(x^2 + 2x) &= 701 \\
 49(y^2 - 2y + 1) - 15(x^2 + 2x + 1) &= 701 + 49 - 15 \\
 49(y-1)^2 - 15(x+1)^2 &= 735 \\
 \frac{49(y-1)^2}{49 \cdot 15} - \frac{15(x+1)^2}{15 \cdot 49} &= \frac{735}{735} \\
 \frac{(y-1)^2}{15} - \frac{(x+1)^2}{49} &= 1
 \end{aligned}$$

Parabola:  $y^2 - 12x - 6y + 33 = 0$

$$\begin{aligned}
 y^2 - 6x &= 12x - 33 \\
 y^2 - 6x + 9 &= 12x - 33 + 9 \\
 (y-3)^2 &= 12x - 24 \\
 (y-3)^2 &= 12(x-2) \\
 \text{Same as} & \\
 \text{before} &
 \end{aligned}$$

To  $\checkmark$  Find a vertex (not the center) or a point on it (Alpha, center, the graph makes this pretty easy) and plug it into the general form.

CV/HW - What's it

#9 is a bit weird  
at the end. ~~go~~  
Go with it.